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New method of fault-distribution-dependent memory-reliable controller design for discrete-time systems with stochastic input delays

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Abstract: A new memory-reliable controller design for a class of discrete-time systems with time-varying input delays is proposed. By assuming that the actuator fault obeys a certain probabilistic distribution, a new practical actuator fault model is presented. Based on this fault model and the known past information, an augmented system with time-varying delay or non-delay, similar to switched systems, is established. By using the Lyapunov–Krasovskii approach, a sufficient condition for the existence of reliable controller is expressed by linear matrix inequalities. An illustrative example is exploited to show the effectiveness of the proposed design procedures.

1 Introduction

It is well known that the time delays are ubiquitous in the control systems. They usually enter because of the sensors and actuators used in them and have been thought to have a deleterious effect on both the stability and the performance of controlled systems, and much research has been done in attempting to eliminate them, compensate for them or nullify their presence. Time delays can be generally classified into two types: systems with state delays [1–5] and systems with input delays [6–10].

From the above-mentioned references, it can be found that most efforts on the memoryless state feedback controller design for continuous-time systems (CTSs) and discrete-time systems (DTSs). However, using the traditional method, sometimes, cannot stabilise the system, as the example in Remark 4, Section 2. As far as we know, few references have been concerned with the memory controller design for DTS with time-varying input delays under an assumption that partial probabilities of input delays are known. In this paper, we employ the partial past controller information and the probabilities of the input delays to address the problem of controller design for DTSs with stochastic input delay.

However, all the aforementioned results are under a full reliability assumption that all actuators are operational. In fact, actuators play a very important role in control system, which are responsible for transforming the controller output to the plant, and how to preserve the closed-loop control system performance under actuator fault condition will be more meaningful. In practical situations, complete failure or partial failure of actuators often occurs. The main task of this study is to design a controller such that the closed-loop system can maintain stability and performance, not only when all control components are operational, but also in case of some existing abnormal actuators including fully outages. To the best of our knowledge, there are very few papers dealing with the reliable control for the DTSs with stochastic actuator failure and input delays. This motivates the development of the so-called reliable control theory.

Over the past few decades, the study of reliable control problems becomes more and more practically meaningful and has attracted considerable attention [11–18]. It is noted that the reliable controller design methods in the aforementioned literatures are all based on the assumption that control component failures are modelled as outages, that is, when a failure occurs, the actuators signal simply becomes zero. However, it cannot represent actuator failure exactly. The actuator may not be complete failure, that is, the scale factor $\xi_i = 0$ is the simplest special cases. In practical systems, because of actuators aging, zero shift, electromagnetic interference, non-linear amplification in different frequency field and so on, it will be more reasonable that the fault-scale factor obeys a certain probabilistic distribution in an interval. To the best of our knowledge, it seems that there are no results on the problem of reliable control with such an actuator fault model that satisfies a certain probabilistic distribution. This motivates us to further investigate the problem of reliable control systems with stochastic input delays and actuator failures.

This paper concerns the problem of the memory-reliable controller design for a class of DTSs with time-varying input delays and stochastic actuator failures. The main contributions of this paper are the following: (i) A more general failure model is presented, which satisfies a certain probabilistic distribution. (ii) By employing the past controller information and the known partial probabilities of the input delays, an augmented system with non-delays (the original system is with constant input delay or with all known probabilities input delays) or time-varying delays (the original system is with partial known probabilities input delays) is established. (iii) Based on those ideas, results for the DTSs with constant input delay and all known probability input delays are extended. Based on those new models, memory-reliable controllers are designed, which can maintain stability and performance, not only when all control components are operational, but also in case of some existing abnormal actuators including fully outages. An illustrative example is exploited to demonstrate the applicability of the proposed design approach.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). When x is a stochastic variable, $\mathcal{E}\{x\}$ stands for the expectation of x. The asterisk $*$ in a matrix is used to denote the term that is induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

2 System description

In this paper, we consider a linear DTS with time-varying input delays described by

$$x(k+1) = Ax(k) + Bu(k - \tau(k)) \tag{1}$$

$$x(k) = \phi(k), \quad k = -\tau_M, \quad -\tau_M + 1, \ldots, 0 \tag{2}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ denote the state vector and the control vector, respectively. A and B are known constant matrices with appropriate dimensions. $\phi(k)$ is the initial condition.

The actuator fault model is described by

$$\begin{aligned} u(k) &= \Xi KX(k) \\ &= \sum_{i=1}^{m} \xi_i C_i KX(k) \end{aligned} \tag{3}$$

where $X(k) = [x^{\mathrm{T}}(k)\, u^{\mathrm{T}}(k-h) \cdots u^{\mathrm{T}}(k-1)]^{\mathrm{T}}$, $\Xi = \mathrm{diag}\{\xi_1 \cdots \xi_m\}$ with $\xi_i (i = 1, \ldots, m)$ being m unrelated random variables. It is assumed that ξ_i is with mathematical expectation μ_i and variance σ_i^2, respectively, and $C_i = \mathrm{diag}\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}$.

Remark 1: There are some open literatures discussing probabilistic sensor failures for the discrete systems. In [19–23], the Bernoulli distributed variable γ is used to describe the sensors failure, wherein $\gamma = 0$ and 1 represent the meaning of completely failure or completely normal. In [24, 25], the authors assumed that the random variables γ taking values in the interval [0, 1], for $0 < \gamma < 1$, it means partial failure. However, a fact has been omitted by most of the researchers that when the sensors/actuators have faults, it may result in backward or forward drift; in this case, the sensors/actuators output might be bigger than the real output, which is very normal in the practical systems; however, it has not caused considerate attention up to now.

Remark 2: Equation (3) describes a phenomenon of actuator drift by a random matrix Ξ satisfying a certain probabilistic distribution in an interval, ξ_i belongs to the interval $[0, \bar{\xi}]$ with $\bar{\xi} \geq 1$. For $\xi_i = 0$, it means complete failure of the ith actuator; for $\xi_i = 1$, it means that the ith actuator is in good working condition; for $0 < \xi_i < 1$, it means partial failure of the ith actuator; for $\xi_i > 1$, it means the actuator-amplifier with forward drift. It should be noted that, in many cases, the gain of actuators could be larger than normal cases by reasons of the surrounding influence or actuator–amplifiers themselves. Therefore the mathematical expectation μ_i of random variance ξ_i, similar to the scaling factor in [26], should be defined as $0 < \mu_i < \bar{\mu}_i$, where $\bar{\mu} \geq 1$. Furthermore, σ_i denotes the gain of actuators' fluctuation levels because of the influence of all the factors acting on actuators.

Remark 3: $\mu_i = \mathcal{E}\{\xi_i\}$ represents the failure rate of the ith actuator. It should be noted that with the consideration of the influence of all the factors, $\mu_i = 1$ does not mean that the ith actuator is always in good working condition; the values of ζ_i can be bigger or smaller than 1. Similarly, $\mu_i = 0$ does not mean in the complete failure of the ith actuator. In particular, If the case $\mu_i = 0$, and $\sigma_i = 0$, simultaneity, it stands for an entire missing of signals, and if $\mu_i = 1$, $\sigma_i = 0$ indicates intactness. In fact, actuator–amplifiers backward or forward drift usually occurs in practice situations, while completely failure and intactness are only two special cases.

Remark 4: The controller we usually used is memoryless, that is, $u(k) = \hat{K}x(k)$; however, in some cases, the system cannot be stabilised whatever the value of the controller gain takes. For example, for system (1), if we select $A = 1, B = 1, u(k) = \hat{K}x(k), \tau(k) = 1$, that is

$$x(k+1) = 3x(k) + \hat{K}x(k-1) \tag{4}$$

Obviously, system (4) cannot be stabilised by using the traditional state feedback method whatever the value of $\hat{K}$ takes, which motivates us to find a novel controller to address this problem. In this paper, the controllers past information are introduced into the new controller (3), which can address this problem.

Owing to the introduction of the augmented vector $X(k)$, the system (1) under the reliable controller (3) can be rewritten as follows: if we take the input delay $\tau(k) = 1, \ldots, h$, and $d_k + h$, where $d_k = \tau(k) - h$ when $\tau(k) \geq h$, respectively

$$\begin{aligned} X(k+1) &= A_1X(k) + B_1u(k) \\ &\vdots \quad \vdots \\ X(k+1) &= A_hX(k) + B_1u(k) \\ X(k+1) &= A_\beta X(k) + B_2X(k-d_k) \end{aligned}$$

where

$$A_i = \begin{bmatrix} A & 0 & \overbrace{B \quad \cdots}^{i} & 0 \\ 0 & 0 & I \quad \cdots & 0 \\ \cdots & \cdots & \cdots \quad \ddots & \cdots \\ 0 & 0 & 0 \quad \cdots & I \\ 0 & 0 & 0 \quad 0 & 0 \end{bmatrix}$$

$$A_\beta = \begin{bmatrix} A & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ I \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & B & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$i = 1 \cdots h$

In this paper, the probability of the random delay taking some value is assumed to be known. In order to employ these information in the system model, the following definition and assumption are needed.

Defining stochastic variables $\alpha_i(k)$ and $\beta(k)$ as

$$\alpha_i(k) = \begin{cases} 1, & \tau_k = i \\ 0, & \tau_k \neq i \end{cases}, \quad i = 1, 2, \ldots, h \tag{5}$$

$$\beta(k) = \begin{cases} 1, & \tau_k > h \\ 0, & \tau_k \leq h \end{cases} \tag{6}$$

Assumption 1: $E\{\alpha_i(k)\} = E\{\alpha_i^2(k)\} = \alpha_i$, $E\{\beta(k)\} = E\{\beta^2(k)\} = \beta$ and $\sum_{i=1}^{h} \alpha_i(k) + \beta = 1$, that is, α_i and β are the probabilities of $P_{\text{rob}}\{\tau(k) = i\}$, $i = 1, 2, \ldots, h$ and $P_{\text{rob}}\{\tau(k) > h\}$.

Then, the system (1) can be rewritten as

$$\begin{aligned} X(k+1) = {} & \sum_{i=1}^{h} \alpha_i(k)\{A_iX(k) + B_1u(k)\} \\ & + \beta(k)\{A_\beta(k)X(k) + B_2X(k-d_k)\} \end{aligned} \tag{7}$$

where $0 \leq d_k = (\tau(k) - h) \leq d_M$.

From the definition of the reliable controller (3), the system (7) can be further rewritten as

$$\begin{aligned} X(k+1) = {} & \left(\sum_{i=1}^{h} \alpha_{ik}A_i + \beta(k)A_\beta + B_1\Xi_0K + B_1(\Xi(k) - \Xi_0)K \right) X(k) \\ & + \beta(k)B_2X(k-d_k) \end{aligned} \tag{8}$$

Remark 5: In most practical systems, the variation range of the delay is large, and the probability of delay taking large values is usually very little. we can obtain the probability distribution of the partial delays with big probability. With this modelling method, the probability distribution of the delay is added to the parameters of the system; thus more information is utilised, which can be expected to reduce the conservatism and obtain a better control effect.

The objective of this study is to develop a reliable controller for the closed-loop system with the stochastic fault model described by (3). For this purpose, the following lemmas and definitions are introduced.

Lemma 1 [27]: Suppose M, N and Ψ are constant matrices of appropriate dimensions. Then

$$(d(k) - d_m)M + (d_M - d(k))N + \Psi < 0 \tag{9}$$

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is true for any $d(k) \in [d_m\ d_M]$, if and only if

$$(d_M - d_m)M + \Psi < 0 \quad (10)$$

$$(d_M - d_m)N + \Psi < 0 \quad (11)$$

3 Main result

The following theorem provides the stability criteria for system (8) with the reliable controller (3).

Theorem 1: For given scalars $\mu_i > 0$ and $\sigma_i > 0$ ($i = 1, \ldots, m$), the system (8) is exponentially mean-square stable, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, N, M with appropriate dimensions such that the following matrix inequalities hold

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \sqrt{d_M}M \\ * & \Omega_{22} & 0 & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -R \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \sqrt{d_M}N \\ * & \Omega_{22} & 0 & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -R \end{bmatrix} < 0 \quad (13)$$

where

$$\Omega_{11} = \begin{bmatrix} \Omega & \beta PB_2 + M_1 - N_1 + N_2^{\mathrm{T}} & -M_1 + N_3^{\mathrm{T}} \\ * & M_2 + M_2^{\mathrm{T}} - N_2 - N_2^{\mathrm{T}} - Q & -M_2 + M_3^{\mathrm{T}} - N_3^{\mathrm{T}} \\ * & * & -M_3 - M_3^{\mathrm{T}} \end{bmatrix}$$

$$\Omega_{12} = \begin{bmatrix} \sqrt{\alpha_1}(A_1 + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R} & \cdots & \sqrt{\alpha_h}(A_h + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R} & \sqrt{\beta}(A_\beta + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R} \\ 0 & \cdots & 0 & \sqrt{\beta}B_2^{\mathrm{T}}\tilde{R} \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\Omega_{22} = \mathrm{diag}\{\underbrace{-\tilde{R}\ -\tilde{R}\ \cdots\ -\tilde{R}}_{h+1}\},\quad \tilde{R} = P + d_MR$$

$$\Omega_{13} = \begin{bmatrix} \sigma_1K^{\mathrm{T}}C_1^{\mathrm{T}}B_1^{\mathrm{T}}\tilde{R} & \cdots & \sigma_mK^{\mathrm{T}}C_m^{\mathrm{T}}B_1^{\mathrm{T}}\tilde{R} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Omega_{33} = \mathrm{diag}\{\underbrace{-\tilde{R}\ -\tilde{R}\ \cdots\ -\tilde{R}}_{m}\}$$

$$\Omega = P\sum_{i=1}^{h}\alpha_iA_i + \sum_{i=1}^{h}\alpha_iA_i^{\mathrm{T}}P + P\beta A_\beta + \beta A_\beta^{\mathrm{T}}P - 2P + PB_1\Xi_0K + (B_1\Xi_0K)^{\mathrm{T}}P + Q + N_1 + N_1^{\mathrm{T}}$$

$$\Xi_0 = \mathrm{diag}\{\mu_1 \cdots \mu_m\}$$

Proof: The Lyapunov functional is constructed as

$$V(k) = X^{\mathrm{T}}(k)PX(k) + \sum_{i=k-d_M}^{k-1}X^{\mathrm{T}}(i)QX(i) + \sum_{i=-d_M}^{-1}\sum_{j=k+i}^{k-1}y^{\mathrm{T}}(j)Ry(j) \quad (14)$$

where $P > 0$, $Q > 0$, $R > 0$ and $y(k) = X(k+1) - X(k)$. Defining $\mathcal{X}(k) = \{X(k), X(k-1), \ldots, X(k-h))\}$, then calculating the difference of $V(k)$ along the system (8) and taking the mathematical expectation, we have

$$\begin{aligned} &\mathcal{E}\{\Delta V(k)|\mathcal{X}(k)\} \\ &= \mathcal{E}\{y^{\mathrm{T}}(k)Py(k)2X^{\mathrm{T}}(k)P \\ &\quad \times \left[\sum_{i=1}^{h}\alpha_iA_i + \beta A_\beta + B_1\Xi_0K - I\right]X(k) \\ &\quad + 2X^{\mathrm{T}}(k)P\beta B_2X(k-d_k) + X^{\mathrm{T}}(k)QX(k) \\ &\quad - X^{\mathrm{T}}(k-d_M)QX(k-d_M) \\ &\quad + d_My^{\mathrm{T}}(k)Ry(k) - \sum_{i=k-d_M}^{k-1}y^{\mathrm{T}}(i)Ry(i)\Bigg\} \end{aligned} \quad (15)$$

Note that

$$\begin{aligned} &\mathcal{E}\{y^{\mathrm{T}}(k)\tilde{R}y(k)\} \\ &= X^{\mathrm{T}}(k)\Bigg\{\sum_{i=1}^{h}\alpha_iA_i^{\mathrm{T}}\tilde{R}A_i + 2\sum_{i=1}^{h}\alpha_iA_i^{\mathrm{T}}\tilde{R}(B_1\Xi_0K - I) + \beta A_\beta^{\mathrm{T}}\tilde{R}A_\beta \\ &\quad + (B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}(B_1\Xi_0K - I) + 2\beta A_\beta^{\mathrm{T}}\tilde{R}(B_1\Xi_0K - I) \\ &\quad + \sum_{i=1}^{m}\sigma_i^2(B_1C_iK)^{\mathrm{T}}\tilde{R}(B_1C_iK)\Bigg\}X(k) \\ &\quad + 2X^{\mathrm{T}}(k)\{\beta(A_\beta + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}B_2\}X(k-d_k) \\ &\quad + X^{\mathrm{T}}(k-d_k)\beta B_2^{\mathrm{T}}\tilde{R}B_2X(k-d_k) \\ &= X^{\mathrm{T}}(k)\sum_{i=1}^{h}\alpha_i(A_i + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}(A_i + B_1\Xi_0K - I)X(k) \\ &\quad + X^{\mathrm{T}}(k)\sum_{i=1}^{m}\sigma_i^2(B_1C_iK)^{\mathrm{T}}\tilde{R}(B_1C_iK)X(k) \\ &\quad + \beta X^{\mathrm{T}}(k)\{(A_\beta + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}(A_\beta + B_1\Xi_0K - I)X(k) \\ &\quad + X^{\mathrm{T}}(k)2(A_\beta + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}B_2X(k-d_k) \\ &\quad + X^{\mathrm{T}}(k)2(A_\beta + B_1\Xi_0K - I)^{\mathrm{T}}\tilde{R}B_2X(k-d_k) \\ &\quad + X^{\mathrm{T}}(k-d_k)B_2^{\mathrm{T}}\tilde{R}B_2X(k-d_k)\} \end{aligned} \quad (16)$$

For matrices N and M with appropriate dimensions, the following equations hold obviously

$$2\zeta^{\mathrm{T}}(k)N\left[X(k)-X(k-d_k)-\sum_{i=k-d_k}^{k-1}y(i)\right]=0 \quad (17)$$

$$2\zeta^{\mathrm{T}}(k)M\left[X(k-d_k)-X(k-d_M)-\sum_{i=k-d_M}^{k-d_k-1}y(i)\right]=0 \quad (18)$$

where $\zeta^{\mathrm{T}}(k)=[X^{\mathrm{T}}(k)X^{\mathrm{T}}(k-d_k)X^{\mathrm{T}}(k-d_M)]$.

$$-2\zeta^{\mathrm{T}}(k)N\sum_{i=k-d_k}^{k-1}y(i)\le d_k\zeta^{\mathrm{T}}(k)NR^{-1}N^{\mathrm{T}}\zeta(k) + \sum_{i=k-d_k}^{k-1}y^{\mathrm{T}}(i)Ry(i) \quad (19)$$

$$-2\zeta^{\mathrm{T}}(k)M\sum_{i=k-d_M}^{k-d_k-1}y(i)\le(d_M-d_k)\zeta^{\mathrm{T}}(k)MR^{-1}N^{\mathrm{T}}\zeta(k) + \sum_{i=k-d_M}^{k-d_k-1}y^{\mathrm{T}}(i)Ry(i) \quad (20)$$

Substituting (16) into (15) and according to (17)–(20), we obtain

$$\begin{aligned}
&\mathcal{E}\{\Delta V(k)|\mathcal{X}(k)\}\\
&\le 2X^{\mathrm{T}}(k)P\left(\sum_{i=1}^{h}\alpha_iA_i+\beta A_\beta+B_1\Xi_0K-I\right)X(k)\\
&\quad+2X^{\mathrm{T}}(k)P\beta B_2X(k-d_k)+X^{\mathrm{T}}(k)QX(k)\\
&\quad+-X^{\mathrm{T}}(k-d_M)QX(k-d_M)\\
&\quad+X^{\mathrm{T}}(k)\sum_{i=1}^{h}\alpha_i(A_i+B_1\Xi_0K-I)^{\mathrm{T}}\tilde{R}(A_i+B_1\Xi_0K-I)X(k)\\
&\quad+2\zeta^{\mathrm{T}}(k)\{N[X(k)-X(k-d_k)]\\
&\quad+M[X(k-d_k)-X(k-d_M)]\}\zeta(k)\\
&\quad+d_k\zeta^{\mathrm{T}}(k)NR^{-1}N^{\mathrm{T}}\zeta(k)\\
&\quad+(d_M-d_k)\zeta^{\mathrm{T}}(k)MR^{-1}N^{\mathrm{T}}\zeta(k)\\
&\quad+X^{\mathrm{T}}(k)\sum_{i=1}^{m}\sigma_i^2(B_1C_iK)^{\mathrm{T}}\tilde{R}(B_1C_iK)X(k)\\
&\quad+\beta X^{\mathrm{T}}(k)\{(A_\beta+B_1\Xi_0K-I)^{\mathrm{T}}\tilde{R}(A_\beta+B_1\Xi_0K-I)X(k)\\
&\quad+X^{\mathrm{T}}(k)2(A_\beta+B_1\Xi_0K-I)^{\mathrm{T}}\tilde{R}B_2X(k-d_k)\\
&\quad+X^{\mathrm{T}}(k-d_k)B_2^{\mathrm{T}}\tilde{R}B_2X(k-d_k)\}
\end{aligned}$$

That is

$$\mathcal{E}\{\Delta V(k)|\mathcal{X}(k)\}\le\zeta^{\mathrm{T}}(k)\Pi\zeta(k)$$

where $\Pi=\Omega_{11}+\Omega_{12}^{\mathrm{T}}\Omega_{22}^{-1}\Omega_{12}+\Omega_{13}^{\mathrm{T}}\Omega_{33}^{-1}\Omega_{13}+d_kNR_2^{-1}N^{\mathrm{T}}+(d_M-d_k)MR_2^{-1}M^{\mathrm{T}}$, by using Lemma 1, we can easily know $\Pi<0$, if

$$\begin{bmatrix}\Omega_{11}+\Omega_{12}^{\mathrm{T}}\Omega_{22}^{-1}\Omega_{12}+\Omega_{13}^{\mathrm{T}}\Omega_{33}^{-1}\Omega_{13} & \sqrt{d_M}M\\ * & -R\end{bmatrix}<0 \quad (21)$$

$$\begin{bmatrix}\Omega_{11}+\Omega_{12}^{\mathrm{T}}\Omega_{22}^{-1}\Omega_{12}+\Omega_{13}^{\mathrm{T}}\Omega_{33}^{-1}\Omega_{13} & \sqrt{d_M}N\\ * & -R\end{bmatrix}<0 \quad (22)$$

hold, by using Schur complement, (12) and (13) are equivalent to (21) and (22), respectively. Similar to [28, 24], we can know

$$\{\Delta V(k)|\mathcal{X}(k)\}<-\lambda_{\min}(\Pi)|\zeta(k)|^2 \quad (23)$$

where $\lambda_{\min}$ is the minimum eigenvalue of Π. Finally, we can confirm that the augmented systems (8) is exponentially mean-square stable. This completes the proof. □

Based on Theorem 1, the gain K of controller (3) can be designed using the following theorem.

Theorem 2: For given scalars $\mu_i>0$ and $\sigma_i>0$ $(i=1,\ldots,m)$, the system (8) with $K=Y\mathcal{X}^{-1}$ is exponentially mean-square stable, if there exist scalars $\rho>0$, and matrix $\mathcal{X}>0$, $\hat{Q}>0$, $\hat{N}$, $\hat{M}$ with appropriate dimensions such that

$$\begin{bmatrix}\hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \sqrt{d_M}\hat{M}\\ * & \hat{\Omega}_{22} & 0 & 0 & 0\\ * & * & -\mathcal{X} & 0 & 0\\ * & * & 0 & \hat{\Omega}_{44} & 0\\ * & * & * & * & -\rho\mathcal{X}\end{bmatrix}<0 \quad (24)$$

$$\begin{bmatrix}\hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \sqrt{d_M}\hat{N}\\ * & \hat{\Omega}_{22} & 0 & 0 & 0\\ * & * & -\mathcal{X} & 0 & 0\\ * & * & 0 & \hat{\Omega}_{44} & 0\\ * & * & * & * & -\rho\mathcal{X}\end{bmatrix}<0 \quad (25)$$

where

$$\hat{\Omega}_{11}=\begin{bmatrix}\hat{\Omega} & \beta B_2\mathcal{X}+\hat{M}_1-\hat{N}_1+\hat{N}_2^{\mathrm{T}} & -\hat{M}_1+\hat{N}_3^{\mathrm{T}}\\ * & \hat{M}_2+\hat{M}_2^{\mathrm{T}}-\hat{N}_2-\hat{N}_2^{\mathrm{T}}-\hat{Q} & -\hat{M}_2+\hat{M}_3^{\mathrm{T}}-\hat{N}_3^{\mathrm{T}}\\ * & * & -\hat{M}_3-\hat{M}_3^{\mathrm{T}}\end{bmatrix}$$

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$$\Omega_{12} = \begin{bmatrix} \sqrt{\alpha_1(1+d_M\rho)}[\mathcal{X}(A_1 - I)^{\mathrm{T}} + Y^{\mathrm{T}}E_0^{\mathrm{T}}B_1^{\mathrm{T}}] & \cdots & \sqrt{\alpha_h(1+d_M\rho)}[\mathcal{X}(A_h - I)^{\mathrm{T}} + Y^{\mathrm{T}}E_0^{\mathrm{T}}B_1^{\mathrm{T}}] \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Omega_{13} = \begin{bmatrix} \sqrt{\beta(1+d_M\rho)}[\mathcal{X}(A_\beta - I)^{\mathrm{T}} + Y^{\mathrm{T}}E_0^{\mathrm{T}}B_1^{\mathrm{T}}] \\ \sqrt{\beta(1+d_M\rho)}\mathcal{X}B_2^{\mathrm{T}} \\ 0 \end{bmatrix}$$

$$\hat{\Omega}_{22} = \mathrm{diag}\{\underbrace{-\mathcal{X} \cdots -\mathcal{X}}_{h}\}$$

$$\hat{\Omega}_{14} = \begin{bmatrix} \sqrt{1+d_M\rho}\sigma_1 Y^{\mathrm{T}}C_1^{\mathrm{T}}B_1^{\mathrm{T}} & \cdots & \sqrt{1+d_M\rho}\sigma_m Y^{\mathrm{T}}C_m^{\mathrm{T}}B_1^{\mathrm{T}} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\hat{\Omega}_{44} = \mathrm{diag}\{\underbrace{-\mathcal{X} \cdots -\mathcal{X}}_{m}\}$$

$$\hat{\Omega} = \sum_{i=1}^{h}\alpha_i A_i\mathcal{X} + \sum_{i=1}^{h}\alpha_i\mathcal{X}A_i^{\mathrm{T}} + \beta A_\beta\mathcal{X} + \beta\mathcal{X}A_\beta^{\mathrm{T}} - 2\mathcal{X} + B_1\Xi_0 Y + Y^{\mathrm{T}}\Xi_0^{\mathrm{T}}B_1^{\mathrm{T}} + \hat{Q} + \hat{N}_1 + \hat{N}_1^{\mathrm{T}}$$

Proof: Defining $\mathcal{X} = P^{-1}$, $R = \rho P$, $\hat{M} = \mathcal{X}M\mathcal{X}$, $\hat{N} = \mathcal{X}N\mathcal{X}$, and $Y = K\mathcal{X}$, pre-, post-multiplying both sides of (12) and (13), respectively with $\mathrm{diag}\{\mathcal{X}, \mathcal{X}, \mathcal{X}, -\Omega_{22}, \mathcal{X}, -\Omega_{44}\}$, we can obtain (24) and (25). This completes the proof. □

By using the similar modelling method, we can also address the problem of the system with constant input delay. Assuming that the input delay is a constant h, then the closed-loop system (1) with the reliable controller (3) can be converted as the system without any time delay, that is

$$\begin{aligned} X(k+1) &= (A_h + B_1\Xi K)X(k) \\ &= [(A_h + B_1\Xi_0 K) + B_1(\Xi - \Xi_0)K]X(k) \end{aligned} \quad (26)$$

Corollary 1: For given scalars $\mu_i > 0$ and $\sigma_i > 0$ $(i = 1, \ldots, m)$ the system (26) with $K = YX^{-1}$ is exponentially mean-square stable, if there exist matrix $X > 0$ with appropriate dimensions such that

$$\begin{bmatrix} -X & XA_h^{\mathrm{T}} + Y^{\mathrm{T}}\Xi^{\mathrm{T}}B_1^{\mathrm{T}} & \sigma_1 Y^{\mathrm{T}}C_1^{\mathrm{T}}B_1^{\mathrm{T}} & \cdots & \sigma_m Y^{\mathrm{T}}C_m^{\mathrm{T}}B_1^{\mathrm{T}} \\ * & -X & 0 & \cdots & 0 \\ * & * & -X & & 0 \\ * & * & * & \ddots & 0 \\ * & * & * & * & -X \end{bmatrix} < 0 \quad (27)$$

Proof: Construct the Lyapunov function

$$V(k) = X^{\mathrm{T}}(k)PX(k) \quad (28)$$

Calculating the difference of $V(k)$ along the system (26) and taking the mathematical expectation, we have

$$\mathcal{E}\{\Delta V(k+1), k)\} = X^{\mathrm{T}}(k)\tilde{\Pi}X(k) \quad (29)$$

where $\tilde{\Pi} = \{(A_h + B_1\Xi_0 K)^{\mathrm{T}}P(A_h + B_1\Xi_0 K) + \sum_{i=1}^{m}\sigma_i^2(B_1C_iK)^{\mathrm{T}}PB_1C_iK - P\}$, By Schur complement, we can know that

$$\begin{bmatrix} -P & (A_h + B\Xi_0 K)^{\mathrm{T}}P & (\sigma_1 C_1 K)^{\mathrm{T}}P & \cdots & (\sigma_m C_1 K)^{\mathrm{T}}P \\ * & -P & 0 & \cdots & 0 \\ * & * & -P & \cdots & 0 \\ * & * & * & \cdots & -P \end{bmatrix} < 0 \quad (30)$$

is equivalent to $\tilde{\Pi} < 0$. Defining $\mathcal{X} = P^{-1}$ and $Y = K\mathcal{X}$, and pre-, post-multiplying both sides of (27) with $\mathrm{diag}\{\underbrace{P, \ldots, P}_{m+2}\}$, we can obtain that $\Pi < 0$. Subsequently

$$\{\Delta V(k)|\mathcal{X}(k)\} < -\tilde{\lambda}_{\min}(\tilde{\Pi})|X(k)|^2 \quad (31)$$

where $\tilde{\lambda}_{\min}$ is the minimum eigenvalue of $\tilde{\Pi}$. That is, filtering systems (26) are exponentially mean-square stable. This completes the proof. □

Next, we will discuss another special case, that is, when the input delay is time varying, but the range of them is small. Furthermore, we can obtain their every probabilities. Then by designing $u(k) = \Xi KX(k)$, where $X(k) = [x^{\mathrm{T}}(k)\ u^{\mathrm{T}}(k-h)\ \cdots\ u^{\mathrm{T}}(k-1)]^{\mathrm{T}}$ and h is the up-bound of the input delay $\tau(k)$, we have

$$\begin{aligned} X(k+1) &= \sum_{i=1}^{h}\alpha_i A_i X(k) + B_1\Xi KX(k) \\ &= \left[\sum_{i=1}^{h}\alpha_i A_i + B_1\Xi_0 K + B_1(\Xi - \Xi_0)K\right]X(k) \end{aligned} \quad (32)$$

Corollary 2: For given scalars $\mu_i > 0$ and $\sigma_i > 0$ $(i = 1, \ldots, m)$ the system (32) with $K = Y\mathcal{X}^{-1}$ is exponentially mean-square stable, if there exist matrix $\mathcal{X} > 0$ with appropriate dimensions such that

$$\begin{bmatrix} -\mathcal{X} & \Upsilon_{12} & \Upsilon_{13} \\ * & -\mathcal{X}_h & 0 \\ * & * & -\mathcal{X}_m \end{bmatrix} < 0 \quad (33)$$

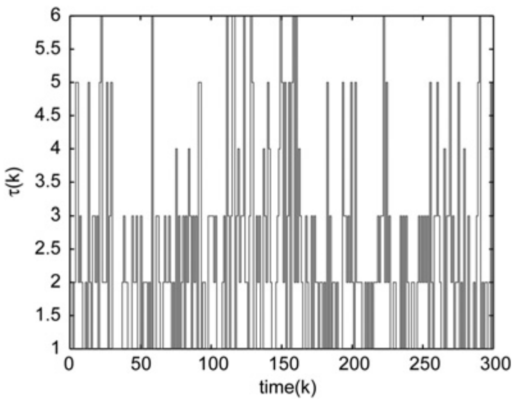

Figure 1 *Stochastic input delay*

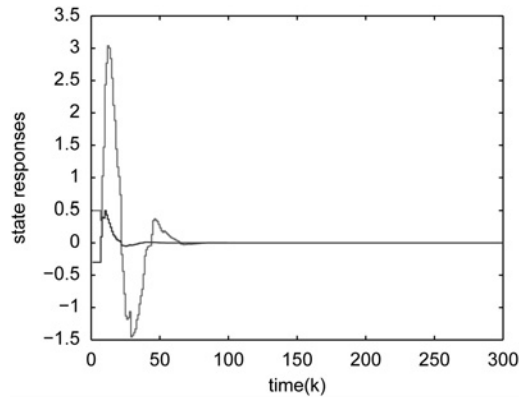

Figure 2 *Standard controller for systems without failure*

where

$$\Upsilon_{12} = [\alpha_1 \mathcal{X} A_1^{\mathrm{T}} + Y^{\mathrm{T}} \Xi_0^{\mathrm{T}} B_1^{\mathrm{T}} \ \cdots \ \alpha_h \mathcal{X} A_h^{\mathrm{T}} + Y^{\mathrm{T}} \Xi_0^{\mathrm{T}} B_1^{\mathrm{T}}]$$

$$\Upsilon_{13} = [\sigma_1 Y^{\mathrm{T}} C_1^{\mathrm{T}} B_1^{\mathrm{T}} \ \cdots \ \sigma_m Y^{\mathrm{T}} C_m^{\mathrm{T}} B_1^{\mathrm{T}}]$$

$$\{\mathcal{X}_{\langle} = \lceil\rangle\dashv\}\{\underbrace{\mathcal{X}\cdots\mathcal{X}}_{\langle}\}$$

$$\mathcal{X}_m = \mathrm{diag}\{\underbrace{\mathcal{X}\cdots\mathcal{X}}_{m}\}$$

Proof: The proof is similar to Corollary 1; this is omitted here. This completes the proof. □

4 Illustrative example

In this section, a well-studied example is used to illustrate the the effectiveness of the approaches proposed in this paper.

Consider the following DTS (1) with the following parameters

$$A = \begin{bmatrix} 1.1 & 2 \\ 0 & 1.02 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 1 \\ -1 & 2 \end{bmatrix}$$

and $1 \le \tau(k) \le 6$, and the initial conditions $x(0) = [-1 \ 1]^{\mathrm{T}}$. Suppose we have known the first 3 delay probabilities, that is, $h = 3$ and $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.2$ and $\beta = 0.2$.

Case 1: We assume that the actuators are normal, that is, the parameter ξ of fault model (3) has expectation $\mu_i = 1$ and variance $\sigma_i = 0$ $(i = 1, 2)$, respectively. According to Theorem 2, we obtain the standard controller K_s (see (34))

Case 2: Assuming that the actuator fault distribution are given by $\Xi_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, that is, there exist partial actuator failure and fluctuations. According to Theorem 2, we obtain the reliable controller K_r (see (35))

The stochastic input delay is depicted in Fig. 1. Figs. 2 and 3 show the state response for the standard fuzzy control design and reliable control, respectively. It is clear that both the controllers perform very satisfactorily when no failures occur. When the actuator is abnormal, which is described by Case 2, the state responses for the standard and the reliable controllers are shown in Figs. 4 and 5, respectively. It is observed that when the actuator fault occur, the closed-loop system with the standard controller is not even asymptotically stable, whereas the closed-loop system using

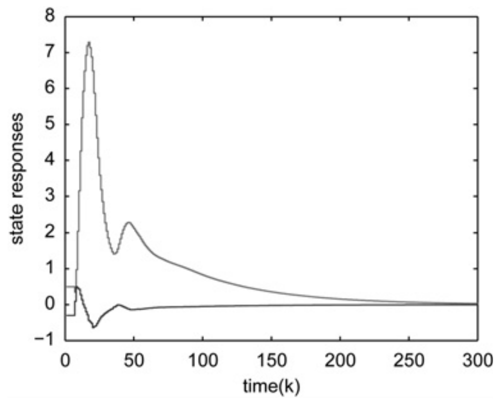

Figure 3 *Reliable controller for systems without failure*

$$K_s = \begin{bmatrix} -0.1490 & -0.9652 & 0.1487 & -0.3265 & 0.3129 & -1.0204 & 0.6936 & -0.4794 \\ -0.0744 & -0.5386 & 0.0773 & -0.1691 & 0.2505 & -0.6992 & 0.1267 & 0.2032 \end{bmatrix} \quad (34)$$

$$K_r = \begin{bmatrix} 0.0008 & 0.0192 & -0.0018 & 0.0038 & -0.0255 & 0.0507 & 0.0645 & -0.1081 \\ -0.0039 & -0.0775 & 0.0067 & -0.0141 & 0.0956 & -0.2023 & -0.1879 & 0.4038 \end{bmatrix} \quad (35)$$

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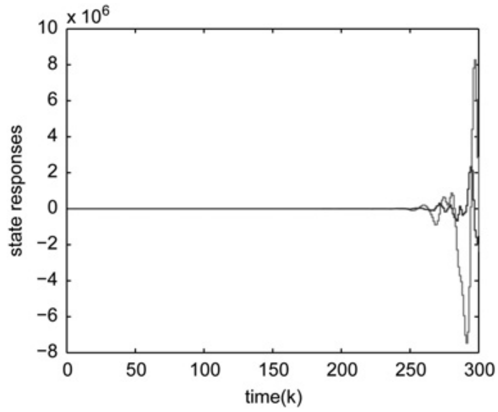

Figure 4 *Standard controller for systems with failure*

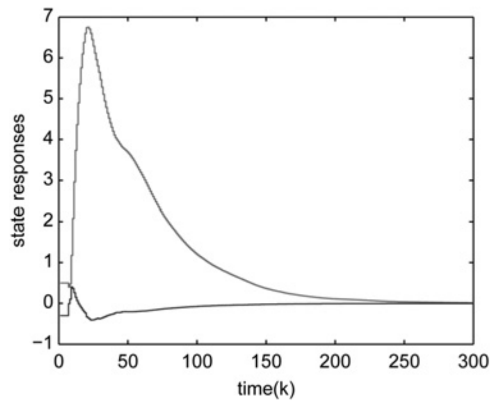

Figure 5 *Reliable controller for systems with failure*

the reliable controller still operates well and maintains an acceptable level of performance.

5 Conclusion

In this paper, a new fault model and design method for the DTSs with input delays are proposed. First, we assume that the actuators fault obeys a certain probability distribution; the stochastic fault model is established. Then, the original system is transformed into an augmented system with non-delays or time-varying delays by employing the past controller information. Based on this new model, reliable controllers are designed to achieve a less conservative result, not only when the system is operating properly, but also in the presence of certain actuator failures. Numerical examples are provided to illustrate the design procedures.

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